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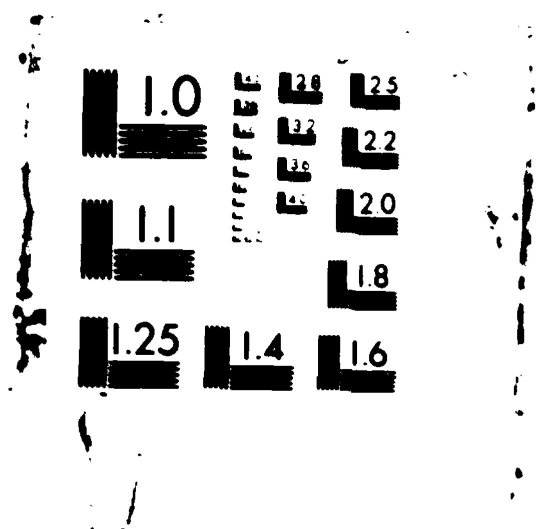

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AD-A179 473

ASSESSING AIRCRAFT SPARES SUPPORT IN A DYNAMIC ENVIRONMENT

Working Note AF401-3

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Randall M. King

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LOGISTICS MANAGEMENT INSTITUTE
6400 Goldsboro Road
Bethesda, Maryland 20817-5886

SUMMARY

Funding to provide for replenishment of reparable spares in the United States Air Force (USAF) is appropriated through Budget Program 1500 (BP-15), the Aircraft Replenishment Spares Program. BP-15 includes the Peacetime Operating Stock (POS) essential to the peacetime readiness goals of the Air Force, as well as the War Reserve Materiel (WRM) needed to sustain forces in a conflict. The requirements for the POS and WRM portions of BP-15 have always been computed separately using analytical techniques that differ widely, making the interrelationship between peacetime readiness and wartime capability hard to understand and quantify accurately.

The Logistics Management Institute's (LMI's) Aircraft Availability Model (AAM) has been used since 1972 by Headquarters USAF in assessing the POS requirement. The AAM is a stochastic, multi-echelon, multi-indenture inventory model that relates the POS portion of BP-15 to a measure of materiel readiness called the "aircraft availability rate." Consistent with its use as a long-range planning tool for peacetime, it is dependent upon a body of "steady-state" inventory theory techniques. This working note describes a recent effort to extend the AAM's capability so that it can assess aircraft availabilities throughout a dynamic conflict scenario.

Because the steady-state mathematics is so deeply embedded in the existing AAM methodology, this working note is necessarily technical. It incorporates recent results in the published literature concerning the theoretical nature of resupply processes in a dynamic environment, as well as approximation techniques for practical use of those results.

The modifications of the AAM described here have been implemented as a prototype in a model referred to as the Surge model. In its current form, the Surge model is an assessment tool only; that is, given a data base of existing stockage levels, it measures the aircraft availability rates for a single weapon system throughout a specific scenario provided by the user. In our judgment, it represents a first step toward a more ambitious objective of developing an integrated model for determining the funding requirements of both the POS and WRM portions of BP-15.

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1. BACKGROUND

The Logistics Management Institute's (LMI's) Aircraft Availability Model (AAM) relates the Peacetime Operating Stock (POS) portion of Budget Program 1500 (BP-15), the Aircraft Replenishment Spares Program, to individual weapon system availability rates.

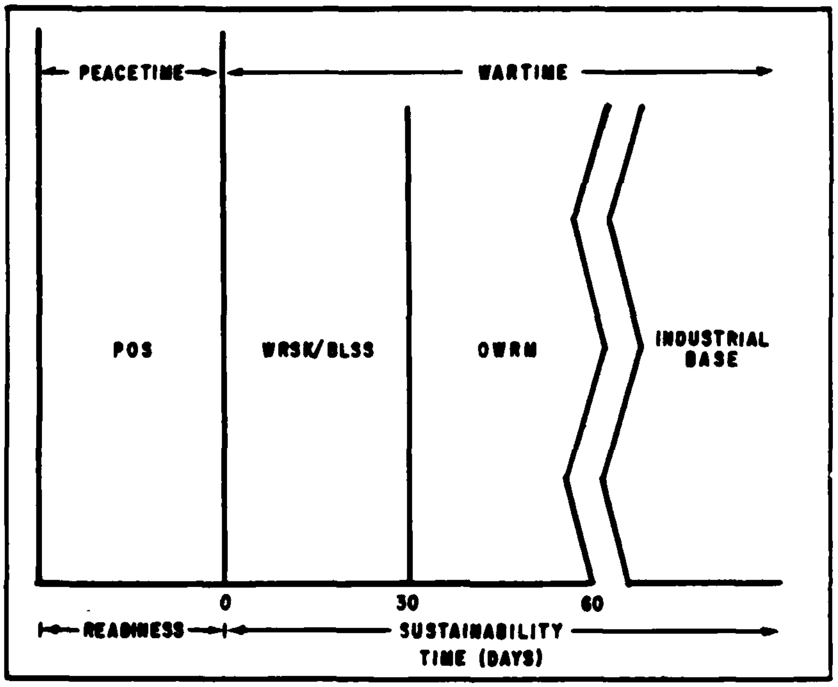
Availability rate is defined as the probability that a randomly chosen aircraft of a particular mission design (MD) is not waiting for a reparable spare. Although the AAM is normally used in Program Objectives Memoranda (POMs) and budget exercises involving the peacetime requirement, this working note documents a recent LMI effort to develop a prototype model for assessing the performance of the supply system under the dynamic requirements of a typical wartime scenario. We begin with an overview of the structure of BP-15.

BP-15 is divided into two major categories: POS and War Reserve Materiel (WRM). POS supports day-to-day operations, the peacetime objectives of training, and provides the materiel readiness necessary in the event of a conflict. WRM funding, on the other hand, provides the additional stockage needed to sustain forces during wartime levels of activity.

WRM is divided between prepositioned and prestocked segments. The War Readiness Spares Kits/Base Level Self-Sufficiency Spares (WRSK/BLSS) represent the prepositioned spares expected to cover the first 30 days of a conflict. WRSK are transportable spares used in support of units to be deployed in the first 30 days; BLSS stocks supplement the peacetime spares for units that engage in place. In addition to the WRSK/BLSS prepositioned segment, WRM includes the prestocked Other War Reserve Materiel (OWRM) segment, which is designed to sustain the forces until the defense industrial base is mobilized.

The components of BP-15 and their roles are summarized in Figure 1-1.

FIGURE 1-1. BP-15 AND READINESS/SUSTAINABILITY OBJECTIVES



SOURCE: FROM FY86 BUDGET ESTIMATE SUBMISSION FOR AIRCRAFT REPLENISHMENT SPARES -- BP-15

The Directorate of Logistics Plans and Programs (LEX) of the Air Staff is responsible for reviewing the total BP-15 requirement during the Planning, Programming, and Budgeting System (PPBS) process. LEX is responsible for both the preparation of the annual Budget Estimate Submission (BES) for BP-15 and the spares portion of the Materiel Readiness Report (MRR) that is submitted each year by the Department of Defense (DoD) to Congress.

LEX manages the BP-15 requirement through the Logistics Capability Measurement System (LCMS). The LCMS is an umbrella system that includes the AAM as its principal tool for analyzing the POS requirement and the Overview model (developed by Synergy, Inc.) for analyzing the WRM requirement.

POS and WRM requirements are managed separately. Nonetheless, there is a clear relationship between existing peacetime stocks and wartime capability.

Moreover, there is an operational sharing of POS and WRM stocks that affects overall supply performance. Spares are drawn from WRSK/BLSS stocks to support the peacetime program. In fact, use of WRM spares in evaluating availability rates is a choice open to the AAM user. (This option, however, is normally not exercised in setting POS requirements.)

LMI, under the sponsorship of the Logistics Concepts Division (LEXY) of LEX, has developed a prototype model for measuring the availability of a specific weapon system that is engaged in a given wartime scenario. The availability rate can be calculated at any point in time in a given scenario and is based on existing inventory levels as recorded in the Recoverable Consumption Item Requirements System (D041) data base. The prototype necessitated substantial modifications of the standard, "steady-state" methodology suitable in the peacetime setting. It also incorporates the effects of cannibalization, because cannibalization represents an acceptable and even essential maintenance policy in wartime.

This working note describes the mathematics of the prototype model and indicates directions of future development. It also includes the major issues involved in extending the prototype from an assessment to a requirements-determination capability for both POS and WRM.

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2. THE SURGE ASSESSMENT MODEL

KEY FEATURES OF THE MODEL

The prototype assessment model incorporates the key features of the AAM. The model:

1. Considers the tradeoffs between stocks at depot and bases. An optimal base/depot distribution is chosen, based on maximization of the weapon system availability rate;

2. Incorporates the levels-of-indenture relationship of the components. Line-replaceable units (LRUs) can contain up to four levels of lower-level shop-replaceable units (SRUs). Shortages of LRUs directly affect availabilities; SRU shortages affect availabilities only indirectly by delaying the repair of the higher-level assemblies; and

3. Treats uncertainty in the projected demands, condemnations, etc. These demands are portrayed as originating from a Poisson process with an estimate for the mean drawn from a Gamma distribution. The resulting distribution for the number in resupply has a negative binomial form that is completely specified by estimates of the mean and variance-to-mean ratio (VMR) parameters. A negative binomial with a VMR value of 1.0 reduces to a Poisson; higher VMR values imply more uncertainty and, consequently, less availability for the same level of funding. (See [9] for more details of the treatment of uncertainty in the AAM.)

The Surge model is fundamentally different from the standard AAM in the mathematics of calculating the various resupply pipelines. (The term "pipeline" is used throughout this working note to denote the mean value of the number of items in a specific resupply process.) The AAM uses the well-known Palm's Theorem [10], which is applied to every instance of resupply in a steady-state situation. In addition, the multi-echelon delay at a base because of depot backorders and the multi-indenture delay of LRU repair because of SRU backorders are dealt with through steady-state mathematics. The dynamic nature of the wartime scenario calls for a new approach that incorporates a dynamic version of Palm's Theorem [4] and a new perspective for the expected backorder (EBO) calculation first developed by Simon [13]. This perspective results in an algorithm that is theoretically exact but computationally difficult. The treatment used in the Surge model is an extension of the approximation technique developed by Slay [14] in a model called VARI-METRIC.

In addition to these fundamental changes in the nature of the pipeline and EBO calculations (which are documented in Chapter 3), the Surge model has been designed to incorporate the effects of maximum cannibalization of LRUs. (It assumes no cannibalization of SRUs.) The model evaluates aircraft availabilities for a single weapon system with a scenario provided by the user.

A typical scenario is shown in Figure 2-1. This scenario resembles those in War and Mobilization Plan (WMP) documents used by wartime planners. The level of intensity usually appears in the WMP in terms of sortie-generation requirements per day. This can be translated into a scenario of flying-hour requirements, given other parameter values, such as sortie length and flying hours per sortie. The approach taken here is to assume a scenario that is already expressed in terms of time-dependent flying-hour requirements; the varying flying-hour program then leads to proportionate changes in failure and condemnation rates.

We have the software developed for the prototype assessment model and implemented it for the C-5A. The results for this weapon system are summarized below.

C-5A RESULTS

The C-5A availabilities are based on the simple step-function scenario portrayed in Figure 2-2. The 65 C-5A aircraft[1] are assumed to generate

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[1]The 65 aircraft represent the aircraft inventory for the second quarter of 1984 as it appears in PA84-3, the Aerospace Vehicles and Flying-Hour Programs System.

FIGURE 2-1. TYPICAL WARTIME PROFILE

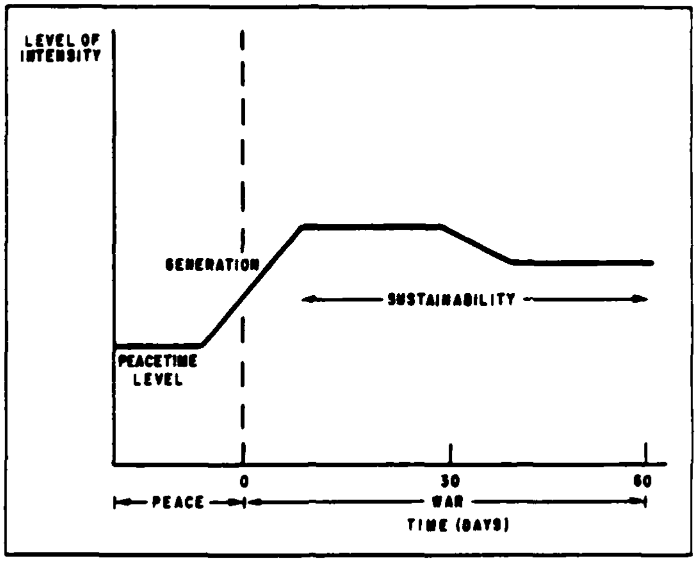
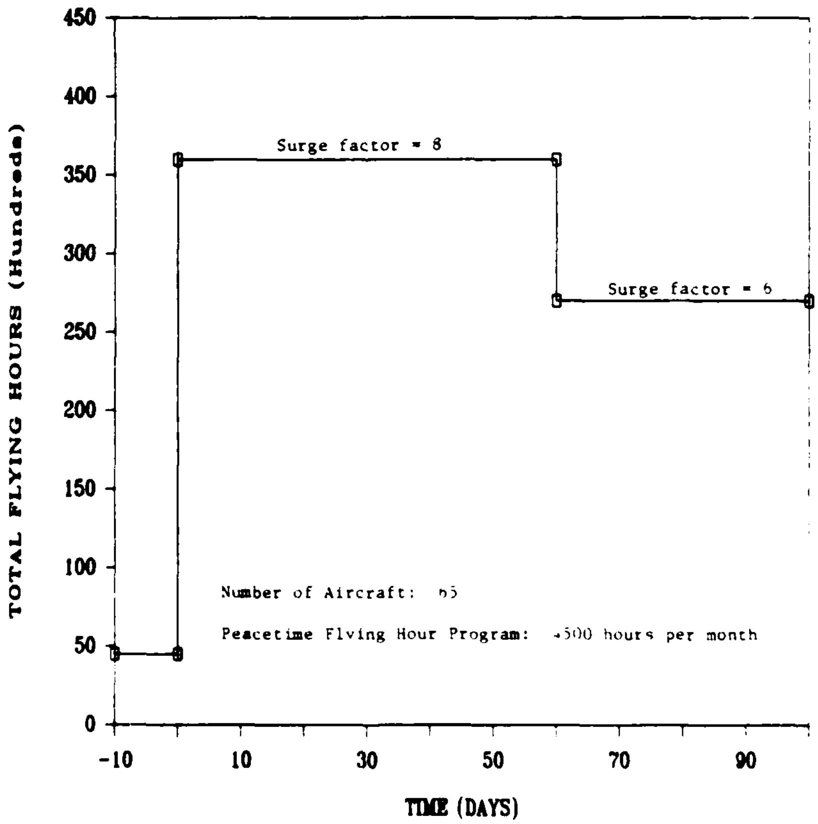
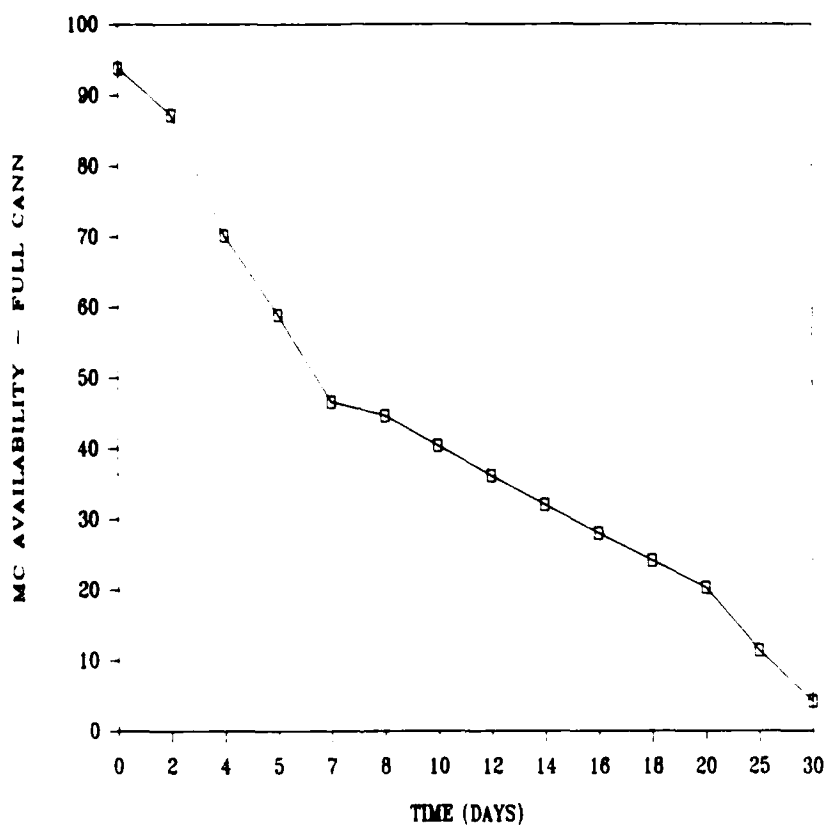


FIGURE 2-2. FLYING-HOUR SCENARIO FOR C-5A



uniform demands at each of three bases. The resulting plot of availability against time for the C-5A is shown in Figure 2-3. These results are termed "mission capable" (MC) in that only those items considered essential to mission performance are included in the computation.

FIGURE 2-3. C-5A RESULTS



In addition, the calculation incorporates cannibalization effects by assuming that LRUs are cannibalized at every base to the maximum extent possible; no cannibalization of SRUs during the repair process is assumed. Figure 2-3 shows that C-5A availability deteriorates quickly. Of course, this specific scenario is especially demanding on the relatively small number of aircraft in the inventory. Some of the other modeling assumptions and processing logic for the C-5A are discussed below with a detailed treatment of the mathematical algorithms to follow in Chapter 3.

The Surge prototype software operates on a single mission design series (MDS). Some preliminary processing was required, therefore, to produce application and demand data for the C-5A alone. In particular, the C-5A "share" of existing assets for common components (components applicable to multiple aircraft types including the C-5A) are prorated on the basis of C-5A usage. The starting asset position (depicted at time T=0 in Figure 2-2) is determined in a manner similar to the near-term availability calculation documented in [6]. All on-order assets are (optimistically) included in the asset position calculation from which Figure 2-3 is derived.

The increased demands resulting from the surge are assumed to impact the Organizational and Intermediate Maintenance (OIM) program only. This will impact the depot with respect to the number of OIM failures that are not base reparable; the Depot Level Maintenance (DLM) programs, by which we mean the regularly scheduled inspection and overhaul of operating components, is, in the prototype model, assumed to continue at the peacetime rate.

In addition to weapon system availabilities, the prototype software identifies those specific components whose poor performance drives the results. For each component, it is possible to compute the expected number of unavailable aircraft (ENUN) resulting from inadequacy in the supply of spares of that component.

Table 2-1 shows ENUN values at various times for five of the poorest performing components. Note that none of these components are problems in the peacetime setting. It is also apparent that a component ranking (in descending order of ENUN) depends on the particular value of T. When cannibalization is incorporated, weapon system availability is driven by a relatively small subset of poorly performing components. On day 30, for example, when approximately 62 of the 65 aircraft are expected to be unavailable. A single component accounts for 57 of them.

TABLE 2-1. PROBLEM COMPONENTS FOR C-5A
SEPTEMBER 1983 DATA BASE

COMPONENT NSN[2]	ENUN[1] ON DAY T			
	T=0	T=20	T=25	T=30
5841001539566LH	0.1	47.1	52.7	57.4
5841001453214LH	0.0	32.2	40.3	48.0
5841000693066LH	0.0	13.1	21.4	32.8
6610005600303	0.0	7.5	18.6	30.7
6605010182181	0.0	16.6	22.5	28.9
C-5A TOTAL	4.0	51.8	57.6	62.3

[1]The ENUN values represent the expected number of unavailable aircraft expected to be unavailable for lack of the given component (despite maximum cannibalization). These values are based on a fleet of 65 C-5A aircraft.

[2]The components listed are associated with the five highest ENUN values at day 30.

The essence of the Surge model logic is the calculation of the expected number in resupply (pipeline) at a particular point in time. We use the Dyna-METRIC approach described in [4] to derive these values. The details of the pipeline calculations, cannibalization logic, and other mathematical algorithms are described in Chapter 3.

3. MATHEMATICAL ALGORITHMS

OVERVIEW OF THE COMPUTATIONAL METHOD

Because the "steady-state" theory is so deeply embedded in the AAM's computational methodology, documenting the changes necessitated by the dynamic environment requires a rather technical discussion. We begin with a brief description of the product formula for availability and the assumptions implicit in that formula and then discuss the incorporation of cannibalization into the availability calculation.

With or without cannibalization, the availability calculation depends on calculation of the backorder distribution of each component. This computation is based on the METRIC formulation by Sherbrooke [11] and Muckstadt's [8] MOD-METRIC extension to handle levels of indenture. In METRIC (and the AAM), each claimant (base or depot) is assumed to follow an (s-1, s) inventory doctrine. In this context, the EBO calculation depends on the evaluation of the inventory position s and the probability distribution function (pdf) for the number in resupply.

In the standard AAM, we develop this pdf by applying Palm's Theorem to each segment (repair or resupply from a higher echelon) of resupply. Palm's Theorem, a classical steady-state result, states that, given demands generated by a Poisson process and resupply times that are independent of demand, the pdf of the number in each resupply segment is itself Poisson and depends on the mean resupply time but not on the specific distribution of the resupply process.

We include in this chapter a detailed examination of the applicability of Palm's Theorem in a simplified but illustrative case. We then present a dynamic version of Palm's Theorem [1, 4] and demonstrate how it is implemented in the surge environment. Last, we summarize the computational modifications required for the EBO calculation in a dynamic environment.

THE PRODUCT FORMULA FOR AVAILABILITY

We define an aircraft as available if it is not missing any reparable components. The availability rate for a particular MD is the percentage of the MD inventory that is available at a specific point in time. The term "availability" is used throughout this working note to mean the probability that an aircraft chosen at random from the MD inventory is available. Thus, if an MD consists of 200 aircraft and has an availability of .75:

$$\text{Expected number of available aircraft} = .75 \times 200 = 150$$

$$\text{Availability rate} = 150/200 \times 100 = 75 \text{ percent.}$$

The AAM computes the MD availability by means of the product formula:

$$A = \Pi_i \; (1 - EBO_i/(NAC \times QPA_i)), \qquad (1)$$

where

EBO_i = expected number of backorders for component i

NAC = number of aircraft in MD inventory.

QPA_i = quantity per application of component i

The product is taken over all the LRUs installed on the aircraft. A more thorough discussion of this formula can be found in [9]. Essentially, we derive it by thinking of EBO_i as the total number of "holes" (component shortages) that are scattered at random across the aircraft within the MD inventory.

Equation (1) represents the evaluation of aircraft availability without consideration of cannibalization. It assumes independence among the individual component backorders. We shall now develop the mathematics for incorporating cannibalization effects into the availability calculation.

CANNIBALIZATION LOGIC

The fact that the AAM does not include cannibalization in its logic is consistent with its use as a long-range budget tool and the widespread belief that cannibalization, withdrawal of WRM assets, and other forms of expedited repair should not be included in the requirements determination process itself. In measuring the ability of the supply system to cope with a specific surge requirement over a relatively short period, however, it is essential that cannibalization effects be included.

As described earlier, Equation (1), for a weapon system availability, is based upon a random scattering of the "holes" (backorders) across the weapon system inventory. The effect of cannibalization is to consolidate these holes upon fewer aircraft than Equation (1) implies. We adopt the approach of assuming that cannibalization is performed to the maximum extent possible on LRUs. For a fixed point in time and a given MD under consideration, define the random variables:

NAVCAN = number of aircraft not available despite maximum cannibalization of LRUs

$NAVCAN_i$ = number of aircraft not available because of shortages of component i.

Then the cumulative distribution function F_{NAVCAN} for the random variable NAVCAN has the form:

$$\begin{aligned} F_{NAVCAN}(n) &= Pr(NAVCAN \le n) \\ &= Pr(MAX_i \{NAVCAN_i\} \le n) \\ &= \Pi_i Pr(NAVCAN_i \le n) \\ &= \Pi_i Pr(BO_i \le n\ QPA_i), \end{aligned} \tag{2}$$

where BO_i and QPA_i represent the number of backorders and quantity per application, respectively, for component i. Evaluation of Equation (2) requires, therefore, some knowledge of the probability distribution of the number of backorders. Assuming this can be done, the expected number of aircraft not available (ENAVCAN) is calculated from Equation (2), since:

$$\begin{aligned} ENAVCAN &= \sum_{n=0}^{\infty} n\ Pr(NAVCAN = n) \\ &= \sum_{n=0}^{\infty} (1 - F_{NAVCAN}(n)). \end{aligned}$$

The corresponding calculation of availability with cannibalization is then:

$$A_{CANN} = (NAC - ENAVCAN)/NAC, \tag{3}$$

where

NAC = number of aircraft in the weapon system inventory.

In summary, the standard availability calculation without cannibalization, as given in Equation (1), depends on the expected number of backorders, whereas availability with the cannibalization formula, as developed in Equations (2) and (3), depends on knowledge of the probability distribution of backorders. We investigate the mathematics of the backorder calculation in the next section.

BACKORDER CALCULATIONS IN THE AAM -- SIMPLIFIED EXAMPLE

The presentation here does not try to replicate the vast literature concerning the METRIC-based theory but does emphasize aspects of the methodology that require modification as we move from steady-state to dynamic considerations. We will concentrate especially on the role of the classic Palm's Theorem [10] in derivation of the steady-state results, the dynamic version of Palm's Theorem, as developed by Hillestad and Carillo [4], and the incorporation of VARI-METRIC [12, 14] techniques into the AAM software in order to analyze the dynamic effects. We begin with a simplified treatment of the METRIC logic.

We assume that the component under consideration fails according to a stationary Poisson process. Notationally, we say that the random variable X is Poisson with mean λ, denoted $X \sim P(\lambda)$, if the pdf of X has the Poisson form:

$$Pr(X=x) = p(x|\lambda) = e^{-\lambda} \lambda^x/x!.$$

We let $p(x|\lambda)$ denote this Poisson probability. The expected value E(X) and variance VAR(X) of X are both equal to the parameter λ. The stochastic process

$$\{X_t = \text{Number of occurrences of some event in an interval of time } t\}$$

is said to be a stationary Poisson process if:

1. The distribution of occurrences depends on the length of the interval and not the end points;

2. Occurrences in non-overlapping time intervals of time are independent; and

3. $X_t \sim P(\lambda t)$; i.e., $Pr(X_t=x) = p(x|\lambda t) = e^{-\lambda t}(\lambda t)^x/x!$.

The parameter λ is referred to here as the "intensity" or demand rate of the process.

Suppose that we have a component in a multi-echelon environment with stocks to be distributed between a single base and depot. Define the following parameters:

λ = demand rate (per day)

r = probability of repair at base

BRT = average base repair time

OST = average order and ship time

DRT = average depot repair time.

We derive the pdf for the number of backorders in the steady-state case. To convey the essence of the calculation without undue complexity, we assume, further, that:

1. All failures are generated at the OIM or base level. The base failures may, with a certain probability, be reparable at the base maintenance activity; otherwise, they are shipped to the depot for possible repair. We are not considering the existence of a separate DLM program, where components are overhauled on a scheduled basis apart from the base failure process;

2. All items shipped to the depot for repair are in fact reparable; i.e., there are no condemnations; and

3. The item has no lower-level subassemblies (SRUs).

The more general treatment is documented at the end of this chapter.

In addition, we assume that each claimant (base or depot) is stocking on an (s-1, s) operating doctrine, where the inventory position s defined by:

s = (quantity on hand) + (quantity due in) - (quantity due out)

is kept constant by replenishment, if necessary, on a one-for-one basis. A backorder exists if the number in resupply (due in) exceeds s. Specifically, the expected number of backorders at any claimant operating under an (s-1, s) policy is given by:

$$E[B(s)] = \sum_{x>s} (x-s)p(x), \tag{4}$$

where

$p(x)$ = the pdf for the number in resupply.

In our simplified example, we develop the methodology for explicitly calculating the pdf $p(x)$ and therefore the expected number of backorders. Moreover, $p(x)$ yields the pdf for backorders needed in the availability with cannibalization Equation (3) because:

$$\Pr(1 \text{ backorder}) = p(s+1)$$
$$\Pr(2 \text{ backorders}) = p(s+2),$$

and, in general:

$$\Pr(k \text{ backorders}) = p(s+k).$$

Consider first the backorders for a random point in time at the depot, given a depot inventory position s_d:

$$E[B_d(s_d)] = \sum_{x>s_d} (x - s_d)p_d(x), \tag{5}$$

where

$p_d(x) = \Pr(x \text{ units in depot repair})$.

Since base demands are Poisson with a demand rate λ and each demand has a probability $1-r$ of being shipped to the depot for repair, the demands at the depot represent a Poisson process with demand rate = $\lambda(1-r)$.

We now appeal to the classic version of Palm's Theorem. Essentially, it says that if demands are Poisson with intensity η, and the resupply process has an arbitrary distribution with mean T with the resupply time independent of demand, the "steady-state" number of items in resupply has a distribution that is itself Poisson with intensity = ηT. (The independence assumption concerning resupply times and demand is, in the queueing theory context, an "infinite-server" assumption. In our setting, it is sometimes referred to as the "slack repair capacity" assumption.) Thus, the pdf $p_d(x)$ in Equation (5) has the form:

$$p_d(x) = p(x \mid \lambda(1-r)\ DRT). \tag{6}$$

We refer to the mean value of a particular resupply process as the pipeline value. Thus, the depot repair pipeline DRPIPE given by:

$$DRPIPE = \lambda(1-r)\ DRT,$$

together with the spares level s_d at the depot, completely specifies the pdf for depot repair, the corresponding pdf for depot backorders, and the expected number of depot backorders given by Equation (5). We apply this result in the derivation of the base backorder distribution that follows.

Equation (4) at the base takes the form:

$$E[B_b(s_b)] = \sum_{x>s_b} (x-s_b)p_b(x), \tag{7}$$

where

$$p_b(x) = \Pr\{x \text{ units in resupply at the base}\},$$

and s_b is the base inventory position. Resupply to the base has two components: (1) the quantities due in from base repair, and (2) items due in from the depot. These resupply processes are independent because the base and depot demands are independent Poisson processes with intensities λr and $\lambda(1-r)$, respectively. In any event, we apply Palm's Theorem to the entire base resupply process. Assuming that successive resupply times are independent of each other and of demand (more on this shortly), it follows that:

$$p_b(x) = p(x|\lambda R),$$

where

$$R = \text{the mean base resupply time.}$$

The problem reduces to finding R. The METRIC result for the mean resupply time at the base is:

$$R = rBRT + (1-r)(OST + \delta(s_d)DRT), \tag{8}$$

where

$$\delta(s_d) = E[B_d(s_d)]/E[B_d(0)].$$

The reasonableness of Equation (8) is easy to justify. With probability r, a given demand will be repaired at the base in which case its expected resupply (repair) time is BRT; otherwise, it is repaired at the depot, in which case the resupply time will be OST plus the delay time at the depot resulting from depot backorders. This depot delay clearly depends on the stock level at the depot and appears in Equation (8) in the form $\delta(s_d)$DRT. The derivation of the $\delta(s_d)$ equation above is based on the classic Little's formula of queueing theory, which states that, under very general conditions, the expected steady-state queue length is the product of the demand rate and the expected waiting time (see [3], for example).

Recalling the hypothesis of Palm's Theorem, Equation (8) is also dependent upon the assumption that the base resupply process is independent of demand. As stated earlier, this is the "slack repair capacity" assumption insofar as the repair processes at base and depot are concerned. The depot delay time is, however, dependent upon demand (unless $s_d = 0$) regardless of any assumptions about repair capacity. For example, the probability that a given demand is immediately filled from depot stocks (in which case the average base resupply time is OST) is obviously dependent upon the recent demand pattern at the depot. The METRIC formulation is then an approximation in that it ignores this correlation. The effect is to understate backorders. (This issue becomes much more significant in the dynamic setting, where the derivation just described of the average base resupply time breaks down completely.)

We summarize below the procedures used in the AAM for calculating the distribution of backorders for a component under our single base hypothesis. The AAM, in the standard implementation, is attempting to compute the weapon system availability that corresponds to an expenditure of BP-15 funds over a specific period of time referred to as the "operating period." Since the impact of monies spent is not felt for a procurement leadtime, the asset positions and various resupply pipeline values for each component are calculated for a point in time that represents an average procurement leadtime beyond the model period.

The AAM first computes a total worldwide asset position (WWASSET) for each component. WWASSET includes all assets projected to be on hand or due in from various sources of resupply (base or depot repair, procurements exogeneous to the supply system, etc.). The AAM then finds the distribution of assets between bases and the depot that maximizes the availability of the weapon system. Moreover, it computes the marginal worth (as a change in availability) that results from each additional procurement so that a curve of cost to availability can be constructed. In our context, we are not interested in the marginal analysis, only in the evaluation of the backorder distribution for this starting asset position WWASSET. In our one base example, the AAM considers each combination (s_b, s_d) such that:

$$s_b + s_d = \text{WWASSET}.$$

Given

λ = (daily) base demand rate

r = probability of repair at base

BRT = average base repair time

OST = average order and ship time (from depot to base)

DRT = average depot repair time,

we have seen that the mean values (pipelines) for the various resupply processes are given by:

DRPIPE	$= \lambda(1-r)\text{DRT}$	depot repair pipeline
BRPIPE	$= \lambda r\ \text{BRT}$	base repair pipeline
OSPIPE	$= \lambda(1-r)\text{OST}$	order and ship pipeline.

The pdf $p_d(x)$ for the number in depot repair is, by Palm's Theorem, Poisson with mean DRPIPE:

$$p_d(x) = p(x|\text{DRPIPE}).$$

From this, we can calculate the EBOs at the depot:

$$E[B_d(s_d)] = \sum_{x>s_d} (x-s_d)p_d(x)$$

and the depot delay function:

$$\delta(s_d) = E[B_d(s_d)]/E[B_d(0)].$$

If we define the depot delay pipeline as:

$$\text{DEPDELAY} = \lambda(1-r)\delta(s_d)\text{DRT},$$

we have, after substituting $E[B(s_d)]/E[B(0)]$ for $\delta(s_d)$ and noting that $E[B(0)] = \text{DRPIPE}$ that $\text{DEPDELAY} = E[B(s_d)]$. Palm's Theorem applied to the entire base resupply process then yields:

$$p_b(x) = p_b(x | \text{BRPIPE} + \text{OSPIPE} + \text{DEPDELAY}).$$

The number of EBOs at the base is, then:

$$E[B_b(s_b)] = \sum_{x>s_b} (x-s_b)p_b(x).$$

Moreover, the entire pdf for base backorders has been determined so that the availability with or without cannibalization can be computed via Equations (3) and (1). Thus, the availability calculation depends upon the calculation of the total asset position WWASSET and the various pipeline values: BRPIPE, OSPIPE, DRPIPE, and DEPDELAY. We now discuss the analogous computations in the dynamic setting.

BACKORDER CALCULATION IN THE SURGE MODEL -- SIMPLIFIED EXAMPLE

In the dynamic setting, we say that the demand process:

$$N_t = \{\text{Number of demands in } [0,t]\}$$

is a nonhomogeneous Poisson process with intensity function $\lambda(t)$, $t>0$, if:

1. Demands in nonoverlapping intervals of time are independent; and
2. The distribution for the number of demands in the interval $[t_0, t_0+t]$ is Poisson with parameter $\Lambda(t_0+t) - \Lambda(t_0)$, where

$$\Lambda(t) = \int_0^t \lambda(u)\, du.$$

Condition (2) can be expressed as:

$$\begin{aligned}\Pr\{N(t_0+t) - N(t_0) = x\} &= p(x \mid \Lambda(t_0+t) - \Lambda(t_0)) \\ &= \exp\{-(\Lambda(t_0+t) - \Lambda(t_0))\}[\Lambda(t_0+t) \\ &\quad - \Lambda(t_0)]^x/x!.\end{aligned} \tag{9}$$

Thus, $\Lambda(t)$ represents the mean (and variance) for the number of demands in the interval $[0,t]$. Note that if $\lambda(t) = \lambda$ for all t, $\Lambda(t) = \lambda t$ and Expression (9) becomes:

$$\begin{aligned}\Pr\{N(t_0+t) - N(t_0) = x\} &= \exp\{-(\lambda(t_0+t)-\lambda t_0)\}[\lambda(t_0+t) - \lambda t_0]^x/x! \\ &= \exp\{-\lambda t\}(\lambda t)^x/x!,\end{aligned}$$

we have the stationary Poisson process described earlier. Hillestad and Carillo [5] demonstrate that Palm's Theorem can be extended in the following sense. If demands occur according to a nonhomogeneous process with intensity function $\lambda(t)$, and if resupply actions are independent of each other and of the demand process, then X_t, the number of items in resupply at time t, is itself a nonhomogeneous Poisson process with intensity $\rho(t)$ given by:

$$\rho(t) = \int_0^t \lambda(u)[1-F(u,t)]\, du, \tag{10}$$

where $F(u,t)$ represents the cumulative distribution function (cdf) of the (possibly nonstationary) resupply process; i.e.:

$$F(s,t) = \Pr\{\text{Resupply action initiated at time } s \text{ will be complete by time } t\}.$$

The above expression for $\rho(t)$ also assumes that the number in resupply at time $t = 0$ is 0, but this is only a notational convenience. Note, however, the key difference between the dynamic and steady-state versions: The average number in resupply at time t, $\rho(t)$, depends <u>explicitly</u>, in the dynamic case, on the distribution of resupply times.

We return now to the hypothesis of our simplified example, a component with OIM-generated failures only -- no SRUs and no condemnations -- whose stock is to be allocated between a single base and a depot. As before, let r be the probability of base repair and assume that base demands are nonhomogeneous Poisson with intensity function $\lambda(t)$. Assume further that all resupply times (BRT, OST, and DRT) are deterministic. The Dynamic Palm's Theorem applied to the depot repair process yields a depot repair pipeline DRPIPE(t) at time t, which, by Equation (10) is:

$$\mathrm{DRPIPE}(t) = (1-r) \int_0^t [1-F_D(u,t)]\lambda(u)\, du,$$

where F_D is the cdf of the depot repair process. But, under the deterministic repair time assumption:

$$F_D(u,t) = \begin{cases} 0 & \text{if } t-u < \mathrm{DRT} \\ 1 & \text{if } t-u \geq \mathrm{DRT}, \end{cases}$$

and the preceding integral becomes:

$$\text{DRPIPE}(t) = \begin{cases} (1-r) \displaystyle\int_{o}^{t} \lambda(u)\, du, \text{ if } t < \text{DRT} \\ (1-r) \displaystyle\int_{t-\text{DRT}}^{t} \lambda(u)\, du, \text{ if } t \geq \text{DRT}. \end{cases} \tag{11}$$

To simplify the notation, assume that the model is in steady state at time $t = 0$; i.e., demands before the start of the surge scenario are stationary Poisson with intensity $\lambda(t) = \lambda_0$ for $t \leq 0$. The steady-state value DRPIPE(0) is then:

$$\text{DRPIPE}(t) = \lambda_0(1-r)\ \text{DRT},\ t \leq 0,$$

and Expression (11) becomes, for an arbitrary time t:

$$\text{DRPIPE}(t) = \int_{t-\text{DRT}}^{t} \lambda(u)\ (1-r)\ du. \tag{12}$$

The kth moment of the backorders at the depot can then be calculated via:

$$E[B_d^k(t)] = \sum_{x>s_d} (x-s_d)^k p(x|\text{DRPIPE}(t)). \tag{13}$$

In particular, the expected value and variance of depot backorders is:

$$E[B_d(t)] = E[B_d^1(t)] \tag{14}$$

$$\text{VAR}[B_d(t)] = E[B_d^2(t)] - (E[B_d(t)])^2. \tag{15}$$

Unfortunately, the steady-state argument for determining EBOs at the base cannot be so easily extended to the dynamic case. There are two reasons:

- The correlation between the depot demands and the depot delay pipelines is more pronounced than in the steady-state case; and
- The previous derivation of the $\delta(s_d)$ expression depended on the steady-state Little's formula. This is now wholly inappropriate.

The following strategy is modeled after the approach taken by Simon [13] and Kruse [7] in developing the exact values for EBOs at the base under steady-state assumptions.

Let $\{X_t : t \geq 0\}$ represent the stochastic process for the number in base resupply at time t. Then, because of the deterministic assumption for OST:

$$X_t = BR_t + DD_{[t-OST,t]} + B_d(t-OST), \tag{16}$$

where

BR_t = number in base repair at time t

$DD_{[t-OST,t]}$ = number of depot demands in [t-OST,t]

$B_d(t-OST)$ = number of depot backorders existing at t-OST.

The three random variables on the right side of Equation (16) are independent by the independent-increment assumption in the original demand process. Moreover, BR_t is Poisson with intensity $\lambda_b(t)$ given by:

$$\lambda_b(t) = r\,\lambda(t),$$

and $DD_{[t-OST,t]}$ is a Poisson random variable with parameter:

$$\int_{t-OST}^{t} (1-r)\,\lambda(u)\,du.$$

Therefore, the expected value and variance of BR_t is given by:

$$\mathrm{BRPIPE}(t) = r \int_{t-\mathrm{BRT}}^{t} \lambda(u)\, du,$$

while the expected value and variance of $DD_{[t-OST,t]}$ is:

$$\mathrm{DEPDEM}(t) = (1-r) \int_{t-\mathrm{OST}}^{t} \lambda(u)\, du.$$

The pdf for depot backorders is known because:

Pr{k depot backorders at time t-OST}

= Pr{s_d + k items in depot repair at time t-OST}

= p(s_d + k DRPIPE(t-OST)).

Thus, the pdf of X_t is the convolution of the three pdfs for BR_t, $DD_{[t-OST,t]}$, and B_d(t-OST). This pdf can be computed exactly as in Kruse [7], but it is difficult computationally. Instead, we propose approximating this pdf in the same way that the VARI-METRIC formulation devised by Slay [14] is used to improve the accuracy of the METRIC-based EBO calculation. Slay and Graves [2] have demonstrated independently that if the pdf for base resupply is modeled as a negative binomial random variable, the resulting EBO calculation is much improved over the METRIC approximation. (The improvement can be quantified because of the exact expressions derived by Kruse.) The pdf for a negative binomial can be expressed in the form:

$$P(x\mu|,Q) = \frac{\Gamma\left(x+\frac{\mu}{Q-1}\right)}{x!\,\Gamma\left(\frac{\mu}{Q-1}\right)} \left(\frac{Q-1}{Q}\right)^{x} \left(\frac{1}{Q}\right)^{\frac{\mu}{Q-1}}, \tag{17}$$

where μ and Q represent the mean and VMR of the pdf and $\Gamma(x)$ is the gamma function satisfying the recurrence relation:

$$\Gamma(n+1) = n\Gamma(n) = n!$$

for integer values of n. Both Slay and Graves derive expressions for the variance of the number in resupply so that the pdf can be parameterized. In our context, the expected value can be written, from Equation (16), as:

$$\begin{aligned} E[X_t] &= E[BR_t] + E[DD_{[t-OST,t]}] + E[B_d(t-OST)] \\ &= BRPIPE(t) + DEPDEM(t) + E[B_d(t-OST)], \end{aligned} \tag{18}$$

and the last term is given by Equation (14). Similarly, because of the independence of the random variables on the right-hand side of Equation (16):

$$\begin{aligned} VAR[X_t] &= VAR[BR_t] + VAR[DD_{[t-OST,t]}] + VAR[B_d(t-OST)] \\ &= BRPIPE(t) + DEPDEM(t) + VAR[B_d(t-OST)], \end{aligned} \tag{19}$$

and the last term is given by Equation (15). Then, $\mu = E[X_t]$ and $Q = VAR[X_t]/E[X_t]$ are the parameter values needed for the pdf Equation (17). We discuss briefly the extension of these ideas to the more general case.

THE EBO CALCULATION -- EXTENSIONS

The preceding section details the modifications in the pipeline calculations required in the dynamic environment so that the EBOs can be computed. The arguments are presented for a component stocked at one base, with no lower-level assemblies, no condemnations, and no DLM-generated failures. We discuss the extension of our EBO methodology to the more general case.

Multiple Bases

If there are NB bases, NB > 1, the depot delay segment of the base resupply process is defined as the portion of the depot backorders that originated from the specific base. The AAM assumes that all bases are uniform with respect to demand and repair rates; the depot delay is therefore the same for all bases, and the average base resupply time given by Equation (8) is unchanged. In the dynamic environment, the decomposition of the base resupply quantity X_t given by Equation (16) is valid so long as $B_d(t)$ is redefined to be:

$B_d(t)$ = number of depot backorders <u>due to a specific base</u>.

Simon [13] has shown that, for a given number of depot backorders, the backorders due to a particular base are binomially distributed with probability f equal to the proportion of the total depot demands originating from that base. The pdf for $B_d(t)$ could be computed explicitly on the basis of this binomial splitting. The VARI-METRIC approach we propose incorporates multiple bases by the following minor modification of Equations (18) and (19):

$$E[X_t] = BRPIPE(t) + DEPDEM(t) + f\ E[B_d(t-OST)] \tag{20}$$

$$VAR[X_t] = BRPIPE(t) + DEPDEM(t) + f\ (1-f)\ E[B_d(t-OST)] + f^2\ VAR[B_d(t-OST)]. \tag{21}$$

These expressions are derived from the well-known formulas for the mean and variance of a binomial distribution and elementary results concerning conditional expectations and variances.

<u>Condemnations</u>

Condemnations at the depot can be portrayed as generating a requirement from a higher echelon, i.e., the commercial vendor. In the steady state, for example, suppose, as before, that a component with a base demand rate λ has probability 1-r of being shipped to the depot for repair. Now suppose there is a positive probability c that the component will be condemned and ordered[1] immediately from the manufacturer. We have a condemnation "pipeline" (mean value), which can be written:

$$\text{CONPIPE} = \lambda(1-r)\ c\ \text{PLT},$$

where PLT denotes the average procurement leadtime. Equation (6) for depot backorders then takes the form:

$$E[B_d(s_d)] = \sum_{x>s_d} (x-s_d)p(x|\text{DRPIPE} + \text{CONPIPE}).$$

In the dynamic case, we get an expression analogous to Expression (13):

$$E[B_d^k(t)] = \sum_{x>s_d} (x-s_d)^k\ p(x|\text{DRPIPE}(t) + \text{CONPIPE}(t)), \tag{22}$$

where

$$\text{CONPIPE}(t) = \int_{t-\text{PLT}}^{t} \lambda(u)\ (1-r)\ c\ du \tag{23}$$

represents the expected condemnations over the interval [t-PLT,t]. Note that the base backorder calculation at time t will depend on the condemnation pipeline value at time t-OST analogously to the argument used in deriving Expression (16).

DLM Demands

The Surge model now treats the three depot level maintenance (DLM) programs as being unaffected by the dynamic scenario; i.e., the DLM program is assumed to continue at the peacetime rate.

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[1]This (s-1, s) assumption is not realistic at the depot. Actually, the condemnations should be treated as reductions in the inventory position. This distinction is insignificant over a scenario whose duration is substantially less than the procurement leadtime. The treatment here allows for uncertainty in the condemnations and is also preferred for purposes of exposition.

This is appropriate because these programs are not generated by base level activity. Consider a component with a constant DLM demand rate τ. We have an additional depot resupply pipeline:

$$\text{DLMPIPE} = \tau \text{ DRT},$$

which must be considered in the depot resupply process. The backorders arising from this DLM activity do not affect the base backorders directly. The normal AAM processing logic handles the allocation of depot backorders to each base. This logic requires no modification in the dynamic environment.

Levels of Indenture

Muckstadt [8] first developed a methodology for the backorder calculation in a multi-echelon and multi-indenture inventory system. Muckstadt's model, called MOD-METRIC, applied the steady-state Little's formula to derive expressions for the additional increment in base repair time of a specific LRU due to a backorder of a constituent subassembly (SRU). Muckstadt's approach was approximate in that he ignored the correlation between the base delay caused by SRU backorders and LRU demand. In any event, the MOD-METRIC approach is not applicable to the dynamic situation, both because of this correlation effect and because of the breakdown of the Little's formula argument. Instead, we apply a VARI-METRIC argument; i.e., we develop expressions for the mean and variance for the number in base resupply (including, of course, the effects of SRU backorders) at any time t and model the base resupply process as a negative binomial distribution with these parameters. Unfortunately, there is no way of quantifying absolutely the improvement in this method, because there is no exact solution available in the multi-echelon, multi-indenture case. Sherbrooke [12] recently developed the most comprehensive paper on the VARI-METRIC method and demonstrated its success by simulation, but he does not consider condemnations or non-OIM demands in his treatment. In the one-base case, the approach is to decompose the number in base repair at time t, X_t, as:

$$X_t = \text{number of LRU demands in } [t\text{-BRT}, t] + \sum_{j=0}^{J} \text{backorders of SRU } j \text{ at time } t.$$

This formulation assumes, as does MOD-METRIC, that every LRU failure corresponds to at most one SRU delay. If the base repair time BRT is treated as deterministic, the terms above are independent processes, since SRU backorders at time t are the result of LRU failures before time t-BRT.[2] In the multi-echelon treatment, it can be shown that the number of SRU demands at the depot resulting from LRU repairs at the depot is binomially distributed; from this, it is possible to derive expressions for the mean and variance of the overall resupply process. Chapter 4 contains a summary of the approach taken in the most general case.

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[2]Strictly speaking, this argument assumes that the BRT for the LRU consists entirely of initial disassembly and fault isolation with negligible time for reassembly. In fact, this assumption also implies that the SRU demands are not "felt" for a length of time equal to the parent LRU base repair time. We will return to this point in Chapter 4.

4. SUMMARY OF DYNAMIC AVAILABILITY CALCULATION

We have seen that the availability calculation depends fundamentally on the calculation of inventory positions (at each base and the depot) and the worldwide EBO computation. The Surge prototype model estimates the total asset position WWASSET at the asset cutoff date of the specific D041 data base by:

WWASSET = on-hand serviceable and unserviceable assets
+ due-in serviceable assets
+ on-order quantities
- due-out quantities.

The presumption in this formulation is that there is enough warning time for the unserviceables to be made ready by the start of the conflict. The inclusion of on-order quantities is an option that was exercised when the results shown in Figure 2-2 were obtained. Included in the "on-hand serviceables" are the spares designated as WRM assets.

Just as in the AAM, the model considers, for a given point in time, all combinations of inventory levels at base and depot, such that:

$$\sum_{i=0}^{NB} s_i = \text{WWASSET},$$

where s_o is the depot level and s_i is the level at base i, $1 \leq i \leq NB$. The model chooses the combination that results in the smallest number of EBOs. Involved in this decision for a given LRU is the allocation of stocks between the bases and depot for the constituent SRUs. The tradeoffs involved in finding the stockage allocations are exactly the same as in the normal AAM logic.

For given stockage levels, we summarize below the methodology for computing the probability distributions for the various resupply process in the most general case, i.e., in the multi-indenture, multi-echelon case with multiple bases, condemnations, and non-OIM demands allowed. The notation is borrowed largely from Sherbrooke [12]. We show the results for two levels of indenture (SRUs and LRUs), but extensions to higher levels are straightforward. We assume that the SRUs themselves are never delayed by shortages of subassemblies, although in practice this assumption need only be made for the lowest level portrayed. (Five levels of indenture are now considered in the AAM and the Surge prototype.)

Consider an LRU that is stocked at NB bases and has J distinct SRUs. Throughout this discussion, the index i = 0, 1, 2, . . ., NB corresponds to the possible "sites" for supply, with i = 0 representing the depot and i = 1, 2, . . ., NB representing any of the NB bases. The subscript j represents either the LRU (j = 0) or any of the constituent SRUs (j = 1, 2, . . ., J). We drop the uniform base assumption; it is not required for this development. Specifically, we define the following parameters:

$\lambda_{ij}(t)$ = Demand rate at time t for component j (j = 0, 1, 2, . . ., J) at site i (i = 0, 1, 2, . . ., NB). The base demand process is assumed to be nonhomogeneous Poisson with intensity $\lambda_{ij}(t)$.

τ_j = DLM demand for component j, j = 0, 1, 2, . . ., J.

r_{ij} = Probability that failure of component j will be repaired at site i.

q_{ij} = Probability that LRU failure at base i is caused by failure of SRU j.

R_{ij} = Deterministic repair time at site i for component j, assuming no delay for repair parts.

T_j = Deterministic order and ship time for component j from depot to any base.

P_j = Deterministic procurement leadtime for component j.

s_{ij} = Inventory position for component j at site i.

Some relationships hold among these parameters: The depot demand rate for the LRU is the sum of the OIM demands originating at the bases and the DLM demands:

$$\lambda_{00}(t) = \sum_{i=1}^{NB} \lambda_{i0}(t)(1-r_{i0}) + \tau_0,$$

while the depot demand rate for the SRU demands resulting from LRU repair is:

$$\lambda_{0j}(t) = \sum_{i=1}^{NB} \lambda_{ij}(t)(1-r_{ij}) + \tau_j.$$

If $X_{ij}(t)$ = number of component j items in resupply for time t at site i, the goal is to determine expressions for the mean and variance of $X_{ij}(t)$, so that the parameters of the presumed negative binomial pdf for X_{ij} can be computed. The EBO calculation then follows.

We first consider the resupply process for SRUs at the depot. Resupply in this context is from two sources: depot repair and replenishment to cover condemnations. These are independent Poisson distributed processes at any instant in time. Thus, for j = 1, 2, . . ., J, we apply the dynamic Palm's Theorem to each resupply process to get:

$$\begin{aligned} E[X_{0j}(t)] = VAR[X_{0j}(t)] &= (\text{depot repair pipeline}) \\ &\quad + (\text{condemnation pipeline}) \\ &= \int_{t-R_{0j}}^{t} \lambda_{0j}(u) r_{0j} \cdot du + \int_{t-P_j}^{t} \lambda_{0j}(u)(1-r_{0j}) du. \end{aligned}$$

Thus, the expected SRU backorders at the depot, $E_t[B(s_{0j})]$, can be determined.

Consider next the LRUs in depot resupply at time t, $X_{00}(t)$.

$X_{00}(t)$ = (depot repair pipeline for LRU at time t)

+ (depot condemnation pipeline for LRU at time t)

+ (SRU_j backorders that are delaying LRU repair at time t).

But,

$$f_{0j}(t) = \lambda_{00}(t)q_{0j}/\lambda_{0j}(t)$$

is the fraction of SRU j depot demands that are needed for LRU repair. For any value $B(s_{0j})$ of backorders, Graves [4] has shown that the number that are delaying LRU repair is distributed binomially with probability parameter $f_{0j}(t)$. Since the SRU fault isolation is assumed to take R_{00} days, the SRU demands "lag" the LRU demands by the parent base repair time. We model this by looking at the SRU backorders at time $t-R_{00}$. Combining this observation with the Graves results, it follows that:

$$\begin{aligned} E[X_{00}(t)] = & \int_{t-R_{00}}^{t} \lambda_{00}(u)r_{00}\,du \\ & + \int_{t-P_0}^{t} \lambda_{00}(u)(1-r_{00})du \\ & + \sum_{j=1}^{J} f_{0j}(t-R_{00})E_{t-R_{00}}[B(s_{0j})] \end{aligned} \tag{25}$$

$$\mathrm{VAR}[X_{00}(t)] = \int_{t-R_{00}}^{t} \lambda_{00}(u) r_{00}\, du \tag{26}$$

$$+ \int_{t-P_0}^{t} \lambda_{00}(u)(1-r_{00}) du$$

$$+ \sum_{j=1}^{J} f_{0j}(t-R_{00})(1-f_{0j}(t-R_{00}))E_{t-R_{00}}[B(s_{0j})]$$

$$+ \sum_{j=1}^{J} f_{0j}(t-R_{00})^2 \mathrm{VAR}_{t-R_{00}}[B(s_{0j})].$$

To obtain the mean and variance for the SRU resupply process at the bases, note that:

$$f_{ij}(t) = \lambda_{ij}(t)(1-r_{ij})/\lambda_{0j}$$

represents the fraction of depot demand for SRU j that is due back to base i. The number of depot SRU backorders delaying resupply to the base is again distributed binomially so that for i = 1, 2, . . ., NB and j = 1, 2, . . ., J:

$$E[X_{ij}(t)] = \int_{t-R_{ij}}^{t} \lambda_{ij}(u) r_{ij}\, du + \int_{t-T_j}^{t} \lambda_{ij}(u)(1-r_{ij}) du \tag{27}$$

$$+ f_{ij}(t-T_j)E_{t-T_j}[B(s_{0j})]$$

$$\mathrm{VAR}[X_{ij}(t)] = \int_{t-R_{ij}}^{t} \lambda_{ij}(u)r_{ij}\,du + \int_{t-T_j}^{t} \lambda_{ij}(u)(1-r_{ij})du \tag{28}$$

$$+ f_{ij}(t-T_j)(1-f_{ij}(t-T_j))E_{t-T_j}[B(s_{0j})]$$

$$+ f_{ij}^{2}(t-T_j)\mathrm{VAR}_{t-T_j}[B(s_{0j})].$$

Finally, we arrive at the corresponding expressions for the LRU resupply process at the base. Let

$$f_{i0} = \lambda_{i0}(1-r_{i0})/\lambda_{00}$$

represent the fraction of depot LRU demand due back to base i. Then,

$$E[X_{i0}(t)] = \int_{t-R_{i0}}^{t} \lambda_{i0}(u)r_{i0}\,du + \int_{t-T_0}^{t} \lambda_{i0}(u)(1-r_{i0})du \tag{29}$$

$$+ f_{i0}(t-T_0)E_{t-T_0}[B(s_{00})]$$

$$+ \sum_{j=1}^{J} E_{t-R_{i0}}[B(s_{ij})].$$

$$\mathrm{VAR}[X_{i0}(t)] = \int_{t-R_{i0}}^{t} \lambda_{i0}(u)r_{i0}\,du + \int_{t-T_0}^{t} \lambda_{i0}(u)(1-r_{i0})du \tag{30}$$

$$+ f_{i0}(t-T_0)(1-f_{i0}(t-T_0))E_{t-T_0}[B(s_{00})]$$

$$+ f_{i0}^{2}(t-T_0)\mathrm{VAR}_{t-T_0}[B(s_{00})]$$

$$+ \sum_{j=1}^{J} \mathrm{VAR}_{t-R_{i0}}[B(s_{ij})].$$

These values are used to parameterize the pdf for $X_{i0}(t)$. The expected LRU backorders at base i are then:

$$E_t[B(s_{i0})] = \sum_{x>s_{i0}} (x-s_{i0})Pr(X_{i0}(t)=x),$$

and the worldwide backorders for the LRU at time t become:

$$EBO = \sum_{i=0}^{NB} E_t[B(s_{i0})].$$

This concludes the discussion of the EBO calculation in the dynamic environment.

LIMITATIONS AND ASSUMPTIONS

Although the existing software handles simple step-function scenarios of the type used in the C-5A implementation, it can be easily modified to handle more general scenarios of the type depicted in Figure 2-1.

The more serious limitations of the prototype model are:

1. Each point plotted in Figure 2-2 is the result of a separate Surge model run, with assets and pipeline values evaluated for that particular day;

2. The model assumes that the flying-hour program requirements of the scenario are always achieved. In the C-5A implementation, therefore, base failures are assumed to occur at eight times the peacetime rate throughout the first 60 days; in executing the scenario, however, the number of available aircraft drops so rapidly that it would be impossible to fly the desired program after a few days; and

3. The current version has not incorporated the VARI-METRIC logic; that is, it ignores the correlation factors between LRU repair and SRU backorders and between base demands and depot backorders. In addition, it makes no correction for the theoretic breakdown of the Little's formula argument for the delay caused by backorders of either higher-echelon or lower-indenture components. There is reason to believe that these errors are much more significant than in the steady-state case.

Because of Problem (1), the present software is rather cumbersome to execute. The user must preselect the days for which the evaluation is desired and fill in the rest of the "curve" by interpolation (portrayed as linear in Figure 2-2). This is only an approximation and a time-consuming process, even for a small data base covering components applicable only to the C-5A. Moreover, there is no linkage of the software from one day to the next. The availabilities for each day are calculated under the assumption of an optimal distribution of spares between bases and depots for that day. This amounts to an instantaneously perfect lateral resupply assumption. The current model must be modified so that the initial (day 0) distribution of assets is passed along through time. It is also not obvious how to determine the initial distribution, because the decision for peacetime levels of activity may be different from the wartime requirement.

Problem (2) can be resolved fairly quickly, but some parameter values are required. Consider the C-5A results. Figure 2-2 is based on a peacetime C-5A flying-hour program of 4,500 hours per month, which is approximately 2.3 hours per plane per day. Under the given scenario, this requirement becomes 18.5 hours per day. Even if aircraft were capable of flying 24 hours per day, 50 of the 65 aircraft would be required in order to fly the daily program; i.e., an availability rate of at least 77 percent is required. This becomes impossible by day 5. This problem can be resolved if the maximum number of flying hours per aircraft per day can be supplied. Of course, this parameter depends on factors exogeneous to the supply system, such as turn times or constraints on crew or maintenance personnel.

Problem (3) must be addressed. Although VARI-METRIC techniques have been coded into some versions of the AAM, they have not been thoroughly tested. Incorporating these techniques is necessary because of the mathematical treatment of the EBO calculation in the dynamic environment.

Finally, the theory outlined in this working note is based on some assumptions. Perhaps the most limiting one is the assumption of deterministic resupply times. Sherbrooke argues that only the order and ship times are subject to this restriction, but it is not clear that this argument holds up in the dynamic setting. This will be an area of future investigation.

The present model also assumes no change in DLM activity during the Surge. We shall also investigate ways to portray the wartime DLM program more realistically.

5. CONCLUSIONS

We have demonstrated the feasibility of modifying the AAM so that it can evaluate aircraft availability rates through time under a given dynamic scenario of activity. The required modifications are not trivial, inasmuch as the AAM relies so heavily on steady-state inventory theory techniques. The algorithms presented in this working note for assessing availability rates in a dynamic environment are derived from a generalization of the Palm's Theorem result concerning the probability distribution of resupply processes and from recent work by Slay [14], Graves [4], and Sherbrooke [12] in improving the accuracy of the EBO calculation using VARI-METRIC techniques.

We have designed prototype software for executing this methodology, which, in its current form, has several deficiencies. In continuing developmental work for the Air Staff, we plan to enhance and improve the current prototype so that it:

- Tracks the base/depot allocation of assets throughout the scenario;
- Measures and adjusts for the infeasibility of a given scenario as availability rates decline; and
- Incorporates VARI-METRIC techniques crucial to accuracy.

We also plan to study ways in which the underlying assumptions on deterministic resupply times can be relaxed and DLM wartime activity can be explicitly modeled.

Finally, we emphasize that, although the work described here is largely evaluative, we believe the ideas documented have potential in a requirements determination sense, i.e., that it is possible to capture the functional relationship between the WRM requirement and sustainability in much the same way as the AAM relates the POS requirement to readiness (as measured by aircraft availability). One of the first questions related to the WRM requirements issue is an appropriate measure of sustainability. We believe much progress is possible in this regard.

This effort represents a promising beginning to the long-range goal of modeling in a unified way the overall peacetime and wartime requirements for spares.

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This report describes the mathematical algorithms of a prototype version of a model designed to relate U.S. Air Force funding for reparable spares to aircraft sustainability measures.

This report documents the calculation of resupply distributions in a dynamic environment and discusses feasible approximations to the exact distribution. This model, in its current form, is used for assessment only, but it contains the mathematical foundation for a comprehensive wartime requirements model.

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